Multiscale and multicontinuum methods

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Multiscale phenomena in heterogeneous media arise in a wide range of applications, including subsurface flow, porous media, composite materials, and reactive transport. A fundamental challenge is to develop coarse-scale models that are both computationally efficient and capable of capturing the essential physical mechanisms occurring across multiple spatial scales. Classical multiscale methods provide accurate numerical approximations of fine-scale solutions, while homogenization and multicontinuum approaches seek effective macroscopic descriptions. Bridging these two remains an important open problem.

In this talk, I will present recent developments in multiscale and multicontinuum modeling based on the Generalized Multiscale Finite Element Method (GMsFEM). The proposed framework establishes a direct connection between multiscale basis functions and multicontinuum representations, enabling the systematic derivation of physically meaningful macroscopic equations from multiscale discretizations. The approach naturally accommodates heterogeneous media with multiple interacting continua, varying numbers of continua across the computational domain, and complex multiscale features.

I will discuss the underlying mathematical ideas, the construction of continuum-dependent multiscale basis functions, and the resulting coarse-scale models. Applications to multiphase flow in heterogeneous porous media will be presented, illustrating how the framework captures interactions between mobile and trapped phases and leads to new effective macroscopic models. The talk will conclude with perspectives on extending these ideas to more general multiscale systems, including evolving continua, coupled multiphysics processes, and hierarchical multiscale modeling